Chapter 13

Horn Antenna

Horn Antenna

- *The horn is widely used as a feed element for large radio astronomy, satellite tracking, and communication dishes found installed throughout the world. In addition to its utility as a feed for reflectors and lenses.*
- *Its widespread applicability stems from its simplicity in construction, ease of excitation, versatility, large gain, and preferred overall performance.*

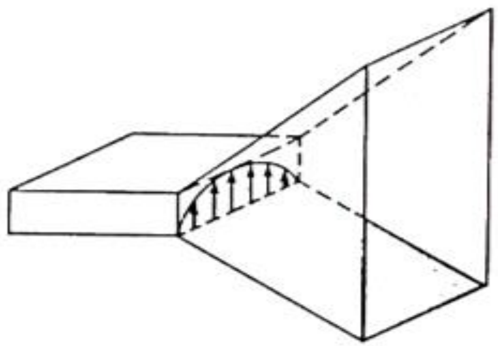
Horn Antenna

- *The horn is nothing more than a hollow pipe of different cross sections, which has been tapered (flared) to a larger opening. The type, direction, and amount of taper (flare) can have a profound effect on the overall performance of the element as a radiator.*

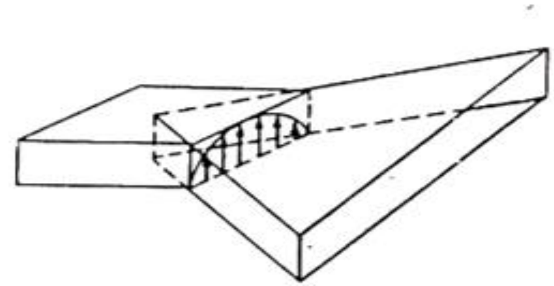
- *An electromagnetic horn can take many different forms, four of which are*

  (a) E-plane *(b) H-plane*
  (c) Pyramidal *(d) Conical*

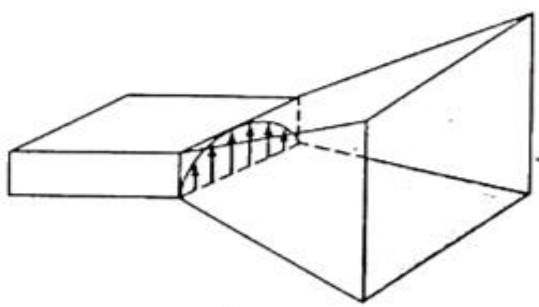
Horn Antenna



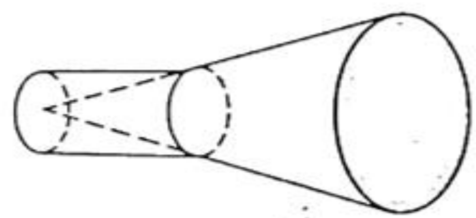
(a) *E*-plane



(b) *H*-plane



(c) Pyramidal



(d) Conical

EM waves that propagate in homogeneous waveguides. This will lead to the concept of "modes" and their classification as

- *Transverse Electric and Magnetic (TEM).*
- *Transverse Electric (TE) .*
- *Transverse Magnetic (TM).*

TE Modes and Rectangular Waveguides

A transverse electric (TE) wave has $E_z = 0$ and $H_z \neq 0$. Consequently, all *E* components are transverse to the direction of propagation.

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad m,n = 0,1,\ldots \quad (m = n \neq 0)$$

For an X-band rectangular waveguide, the cross-sectional dimensions are a = 2.286 cm and b = 1.016 cm. Using (14):

$TE_{m,n}$ Mode Cutoff Frequencies

m	n	$f_{c,mn}$ (GHz)
1	0	6.562
2	0	13.123
0	1	14.764
1	1	16.156

In the X-band region (≈ 8.2-12.5 GHz), only the TE_{10} mode can propagate in the waveguide regardless of how it is excited.

- *This is called single mode operation and is most often the preferred application for hollow waveguides.*
- *On the other hand, at 15.5 GHz any combination of the first three of these modes* ***could*** *exist and propagate inside a metal.*

- **TM Modes and Rectangular Waveguides**

Conversely to TE modes, transverse magnetic (TM) modes have $E_z \neq 0$ and $H_z = 0$.

- The expression for the cutoff frequencies of TM modes in a rectangular waveguide :

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad m,n = 1,2\ldots$$

- It can be shown that if **either** $m=0$ or $n=0$ for TM modes, then $E=H=0$. This means that no TM modes with $m=0$ or $n=0$ are allowable in a rectangular waveguide.

$TM_{m,n}$ Mode Cutoff Frequencies

m	n	$f_{c,mn}$ (GHz)
1	1	16.156
1	2	30.248
2	1	19.753

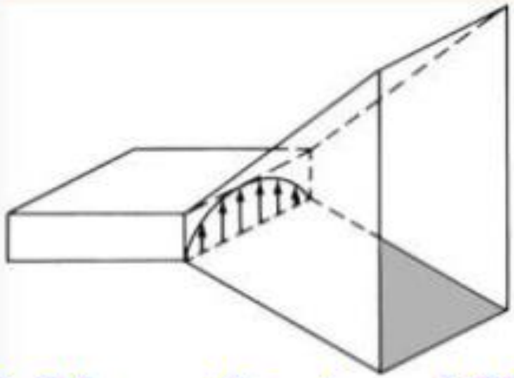
Therefore, no TM modes can propagate in an X-band rectangular waveguide when $f < 16.156$ GHz.

- **Dominant Mode**

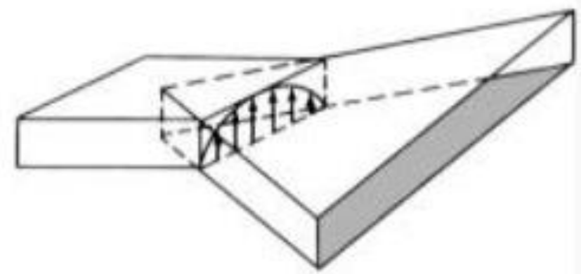
Note that from 6.56 GHz $\le f \le$ 13.12 GHz in the X-band rectangular waveguide, only the TE_{10} mode can propagate. This mode is called the dominant mode of the waveguide.

- **The rectangular horns** are ideally suited for rectangular waveguide feeds. When the feed is a cylindrical waveguide, the antenna is usually a ***conical horn***.
- Why is it necessary to consider the horns separately instead of applying the theory of waveguide aperture antennas directly? It is because the so-called ***phase error*** occurs due to the difference between the lengths from the center of the feed to the center of the horn aperture and the horn edge. This makes the uniform-phase aperture results invalid for the horn apertures.

Horn Antennas

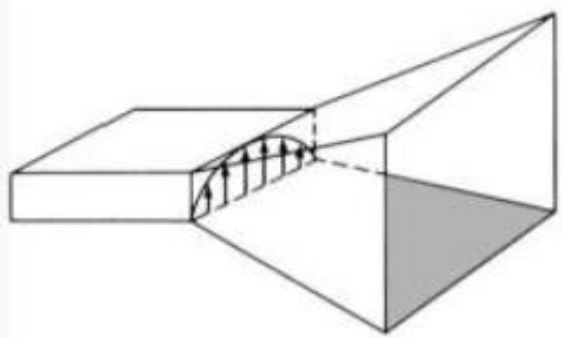


E-Plane Sectoral Horn

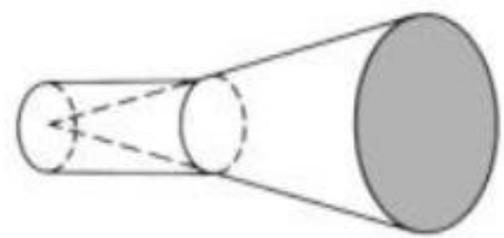


H-Plane Sectoral Horn

TE_{10} mode in Rectangular Waveguide



Pyramidal Horn

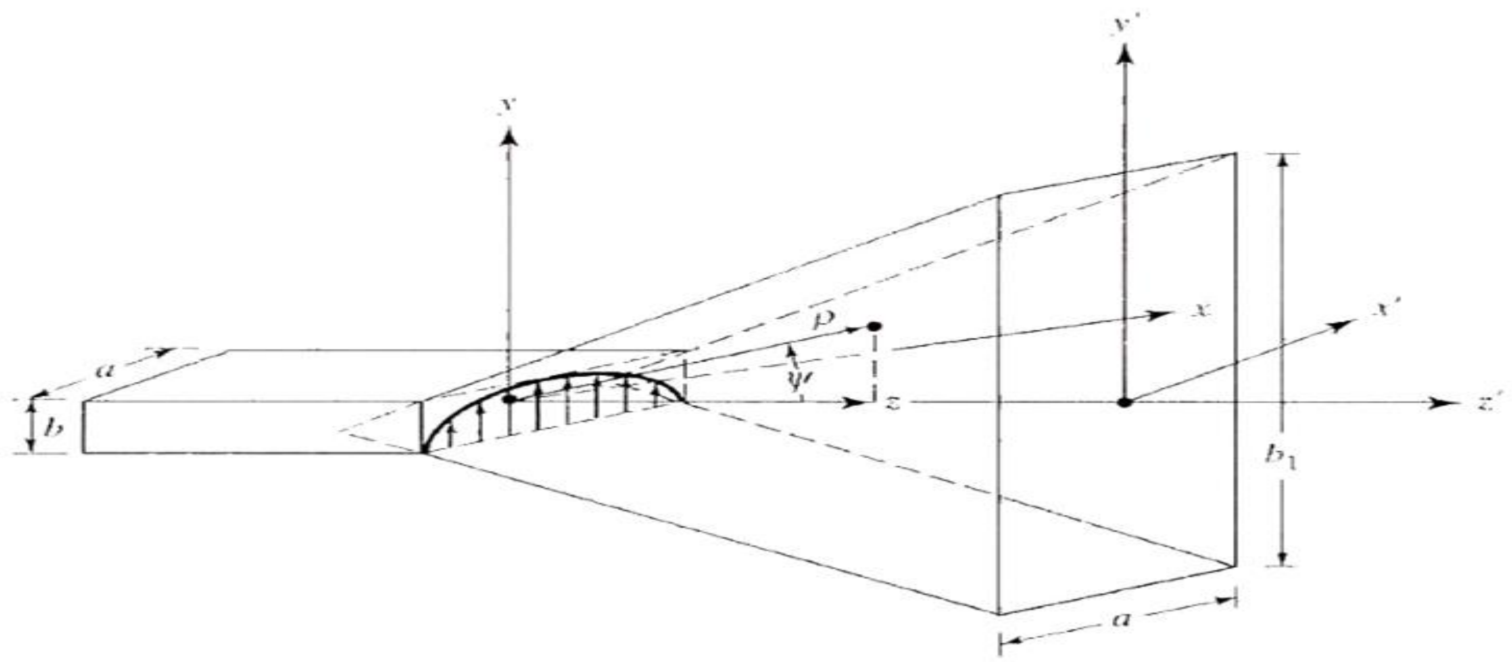


Conical Horn

E Plane Horn

E-Plane Horn

The E-plane sectoral horn is one whose opening is flared in the direction of the E-field.



Aperture Phase Distribution

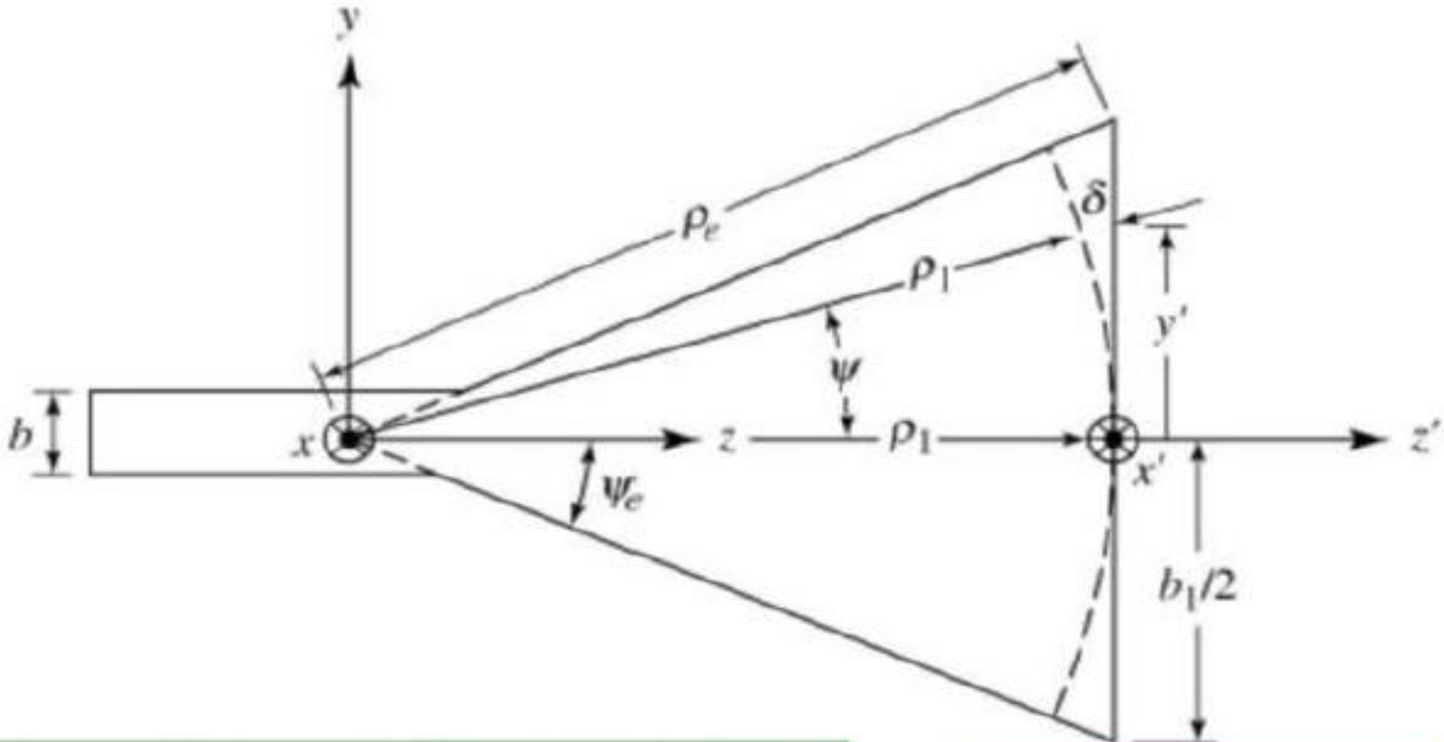
- It is assumed that, there exist a line source radiating cylindrical waves at the imaginary apex of the horn. As waves travel in the outward radial direction, the constant phase fronts are cylindrical which do not coincide with aperture plane.

- At any point y′ at the aperture of the horn, the phase of the field will not be the same as that at the origin.

- The phase difference because the wave travelled different distances from the apex to the aperture.

- The difference in path of travel, designated as δ(y′), can be obtained as follows

E-Plane View



$$\delta(y') = -\rho_1 + \rho_1\left[1 + \left(\frac{y'}{\rho_1}\right)^2\right]^{1/2}$$

$$\Rightarrow \delta(y') \approx \frac{1}{2}\left(\frac{y'^2}{\rho_1}\right)$$

Aperture Phase Distribution

$$\rho_1^2 + (y')^2 = [\rho_1 + \delta(y')]^2$$

$$\rho_1 + \delta(y') = (\rho_1^2 + y'^2)^{1/2} = \rho_1\left[1 + \left(\frac{y'}{\rho_1}\right)^2\right]^{1/2}$$

$$\delta(y') = -\rho_1 + \rho_1\left[1 + \left(\frac{y'}{\rho_1}\right)^2\right]^{1/2}$$

← Exact/Spherical

$$\delta(y') \simeq -\rho_1 + \rho_1\left[1 + \frac{1}{2}\left(\frac{y'}{\rho_1}\right)^2\right] \simeq -\rho_1 + \rho_1 + \frac{1}{2}\frac{y'^2}{\rho_1}$$

$$\delta(y') = \frac{1}{2}\left(\frac{y'^2}{\rho_1}\right)$$

← Quadratic Distance Variation

Aperture Fields

- When $\delta(y')$ is multiplied by the phase constant k, the result is a quadratic phase variation between the constant phase surface and the aperture plane.

Example 13.1

Design an E-plane sectoral horn so that the maximum phase deviation at the aperture of the horn is 56.72°. The dimensions of the horn are $a = 0.5\lambda$, $b = 0.25\lambda$, $b_1 = 2.75\lambda$.

Solution

$$\Delta\phi|_{\max} = k\delta(y')|_{y'=b_1/2} = \frac{k(b_1/2)^2}{2\rho_1} = 56.72\left(\frac{\pi}{180}\right)$$

or

$$\rho_1 = \left(\frac{2.75}{2}\right)^2 \frac{180}{56.72}\lambda = 6\lambda$$

The total flare angle of the horn should be equal to

$$2\psi_e = 2\tan^{-1}\left(\frac{b_1/2}{\rho_1}\right) = 2\tan^{-1}\left(\frac{2.75/2}{6}\right) = 25.81^\circ$$

Aperture Fields

- So the aperture fields become…..

$$E'_y(x',y') = E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j\frac{k}{2}\frac{y'^2}{\rho_1}}$$

Amplitude Distribution ($E_1 \cos\left(\frac{\pi}{a}x'\right)$)

Phase Distribution ($e^{-j\frac{k}{2}\frac{y'^2}{\rho_1}}$)

$$E'_y(x',y') = E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j\frac{k}{2}\frac{y'^2}{\rho_1}}$$

$$H'_x(x',y') = -\frac{E_1}{\eta} \cos\left(\frac{\pi}{a}x'\right) e^{-j\frac{k}{2}\frac{y'^2}{\rho_1}}$$

Far Field Components

$$E_\theta = -j\frac{a\sqrt{\pi k \rho_1}E_1 e^{-jkr}}{8r}\cdot\left\{e^{j\frac{k_y^2\rho_1}{2k}}\sin\phi(1+\cos\theta)\left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2-\left(\frac{\pi}{2}\right)^2}\right]F(t_1,t_2)\right\}$$

$$E_\phi = -j\frac{a\sqrt{\pi k \rho_1}E_1 e^{-jkr}}{8r}\cdot\left\{e^{j\frac{k_y^2\rho_1}{2k}}\cos\phi(1+\cos\theta)\left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2-\left(\frac{\pi}{2}\right)^2}\right]F(t_1,t_2)\right\}$$

where, $k_x = k\sin\theta\cos\phi$

$k_y = k\sin\theta\sin\phi$

Radiation Equations

where

$$F(t_1, t_2) = [C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)]$$

$$t_1 = \sqrt{\frac{1}{\pi k \rho_1}}\left(-\frac{kb_1}{2} - k_y \rho_1\right)$$

$$t_2 = \sqrt{\frac{1}{\pi k \rho_1}}\left(\frac{kb_1}{2} - k_y \rho_1\right)$$

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$$

E Plane Φ = $\pi/2$

$$E_\theta = -j\frac{a\sqrt{\pi k \rho_1}E_1 e^{-jkr}}{8r}$$

$$\cdot\left\{-e^{+j\left(\frac{k\rho_1\sin^2\theta}{2}\right)}\left(\frac{2}{\pi}\right)^2(1+\cos\theta)F(t'_1,t'_2)\right\}$$

$$t'_1 = \sqrt{\frac{k}{\pi\rho_1}}\left(-\frac{b_1}{2}-\rho_1\sin\theta\right)$$

$$t'_2 = \sqrt{\frac{k}{\pi\rho_1}}\left(+\frac{b_1}{2}-\rho_1\sin\theta\right)$$

$$E_r = E_\phi = 0$$

H Plane Φ = 0

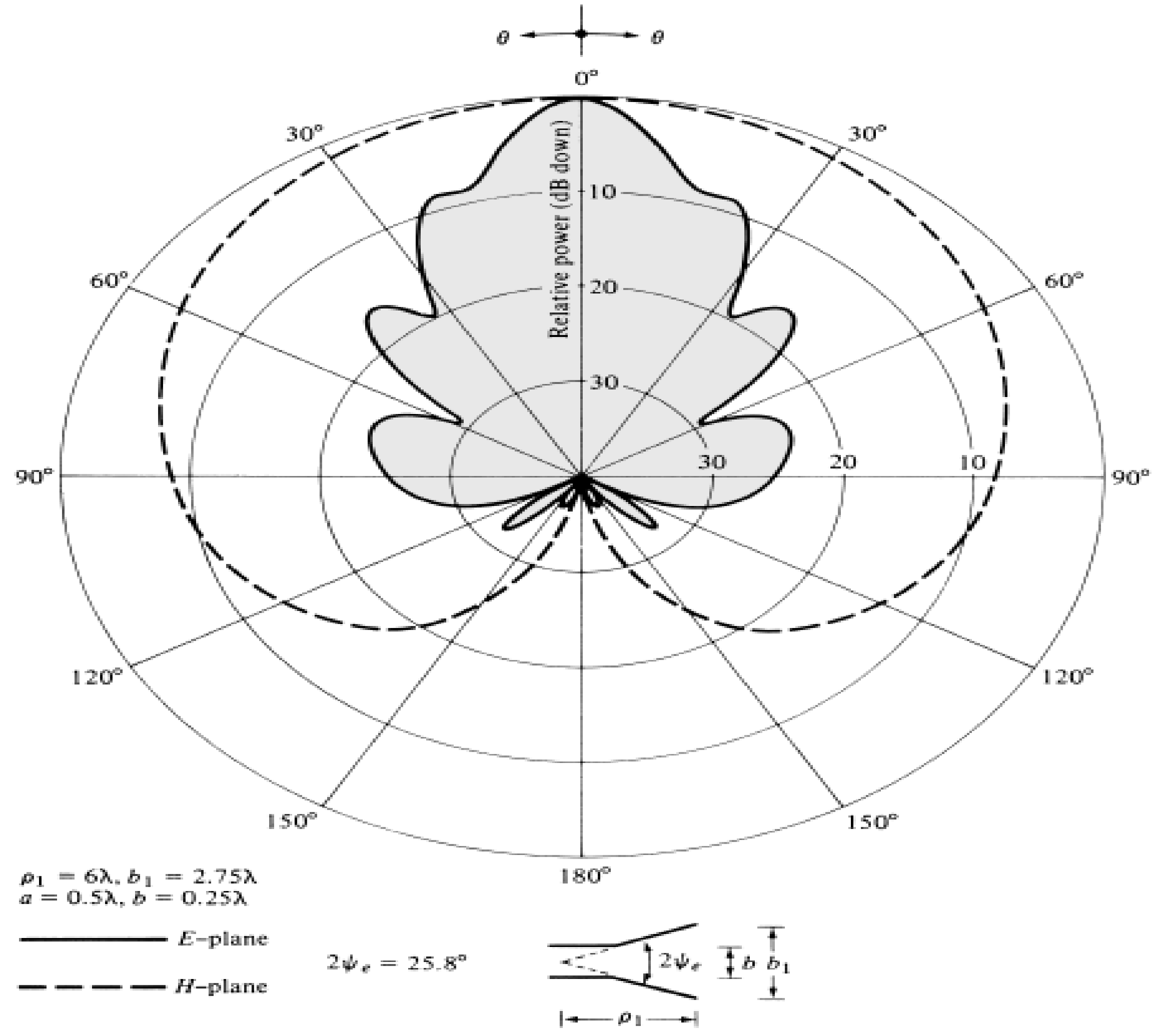
$$E_\phi = -j\frac{a\sqrt{\pi k \rho_1}E_1 e^{-jkr}}{8r}$$

$$\cdot\left\{(1+\cos\theta)\left[\frac{\cos\left(\frac{ka}{2}\sin\theta\right)}{\left(\frac{ka}{2}\sin\theta\right)^2-\left(\frac{\pi}{2}\right)^2}\right]F\left(t''_1, t''_2\right)\right\}$$

$$t''_1 = -\frac{b_1}{2}\sqrt{\frac{k}{\pi\rho_1}}$$

$$t''_2 = +\frac{b_1}{2}\sqrt{\frac{k}{\pi\rho_1}}$$

E-and H-plane patterns of an E-plane sectoral horn



$\rho_1 = 6\lambda$
$b_1 = 2.75\lambda$
$a = 0.5\lambda$

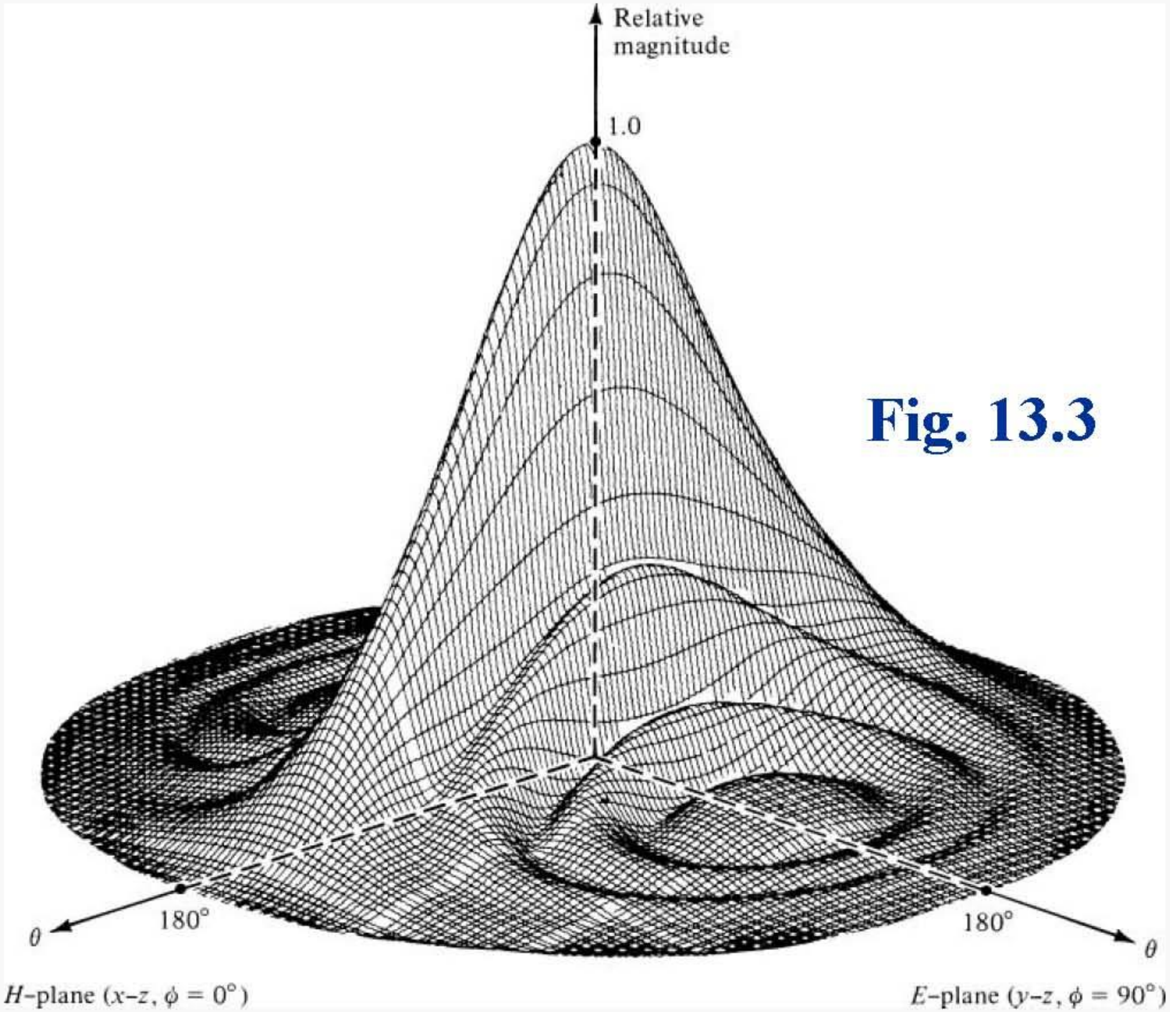


Fig. 13.3

- the *E*-plane pattern is much narrower than the *H*-plane because of the flaring and larger dimensions of the horn in that direction.

Universal Curves

E Plane $\Phi = \pi/2$

Magnitude of Normalized Pattern, Excluding $(1+\cos\theta)$,

can be written as

$$E_{\theta n} = F(t'_1, t'_2) = [C(t'_2) - C(t'_1)] - j[S(t'_2) - S(t'_1)]$$

$$t'_1 = \sqrt{\frac{k}{\pi\rho_1}}\left(-\frac{b_1}{2} - \rho_1 \sin\theta\right)$$

$$= 2\sqrt{\frac{b_1^2}{8\lambda\rho_1}}\left[-1 - \frac{1}{4}\left(\frac{8\rho_1\lambda}{b_1^2}\right)\left(\frac{b_1}{\lambda}\sin\theta\right)\right]$$

$$t'_1 = 2\sqrt{s}\left[-1 - \frac{1}{4}\left(\frac{1}{s}\right)\left(\frac{b_1}{\lambda}\sin\theta\right)\right]$$

$$t'_2 = 2\sqrt{s}\left[+1 - \frac{1}{4}\left(\frac{1}{s}\right)\left(\frac{b_1}{\lambda}\sin\theta\right)\right]$$

$$s = \frac{b_1^2}{8\lambda\rho_1}$$

E Plane $\Phi = \pi/2$

- For a given value of s, the field $E_{\theta n}$ can be plotted as a function of $b_1 / \lambda \sin\theta$, as shown in the following figure for s = 1/64, 1/8, 1/4, 1/2, 3/4, and 1.

- These plots are usually referred to as universal curves, because from them the normalized E-plane sectoral horn can be obtained.

- Finally the value of $(1+\cos\theta)$, normalized to 0 dB and written as $20 \log_{10} ((1+\cos\theta)/2)$, is added to that number to arrive at the required field strength.

Additional Factor To Add To Values Obtained From Universal Curves

$$20\ \ell og_{10}\left[\left(\frac{1+\cos\theta}{2}\right)\right]$$

E Plane Universal Patterns for E Plane Sectoral Horn

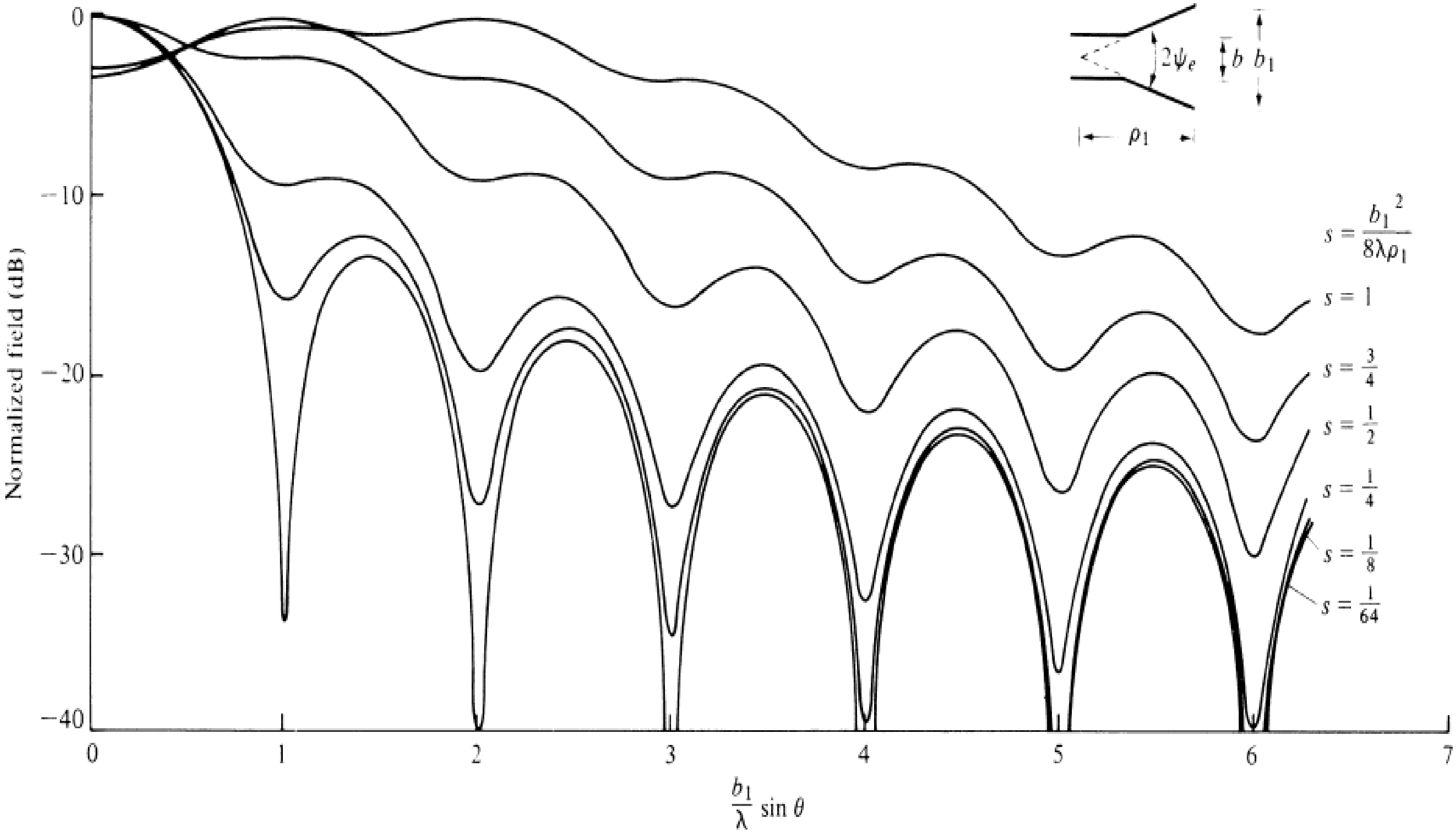


Figure 13.6 *E*-plane universal patterns for *E*-plane sectoral and pyramidal horns.

Example 13.2

An E-plane horn has dimensions of $a = 0.5\lambda$, $b = 0.25\lambda$, $b_1 = 2.75\lambda$, and $\rho_1 = 6\lambda$. Find its E-plane normalized field intensity (*in dB* and *as a voltage ratio*) at an angle of $\theta = 90^\circ$ using the universal curves of Figure 13.6.

Solution: Using (13-14d)

$$s = \frac{b_1^2}{8\lambda\rho_1} = \frac{(2.75)^2}{8(6)} = 0.1575 \simeq \frac{1}{6.3}$$

None of the curves in Figure 13.6 represents $s = \frac{1}{6.3}$. Therefore interpolation will be used between the $s = \frac{1}{4}$ and $s = \frac{1}{8}$ curves.

At $\theta = 90^\circ$

$$\frac{b_1}{\lambda}\sin(\theta) = 2.75\sin(90^\circ) = 2.75$$

and at that point the field intensity between the $s = \frac{1}{4}$ and $s = \frac{1}{8}$ curves is about -20 dB. Therefore the total field intensity at $\theta = 90^\circ$ is equal to

$$E_\theta = -20 + 20\log_{10}\left(\frac{1+\cos 90^\circ}{2}\right) = -20 - 6 = -26 \text{ dB}$$

or as a normalized voltage ratio of

$$E_\theta = 0.05$$

Directivity

1.Calculate B

$$B = \frac{b_1}{\lambda}\sqrt{\frac{50}{\rho_e/\lambda}}$$

2. Using B, find G_E from the following Figure

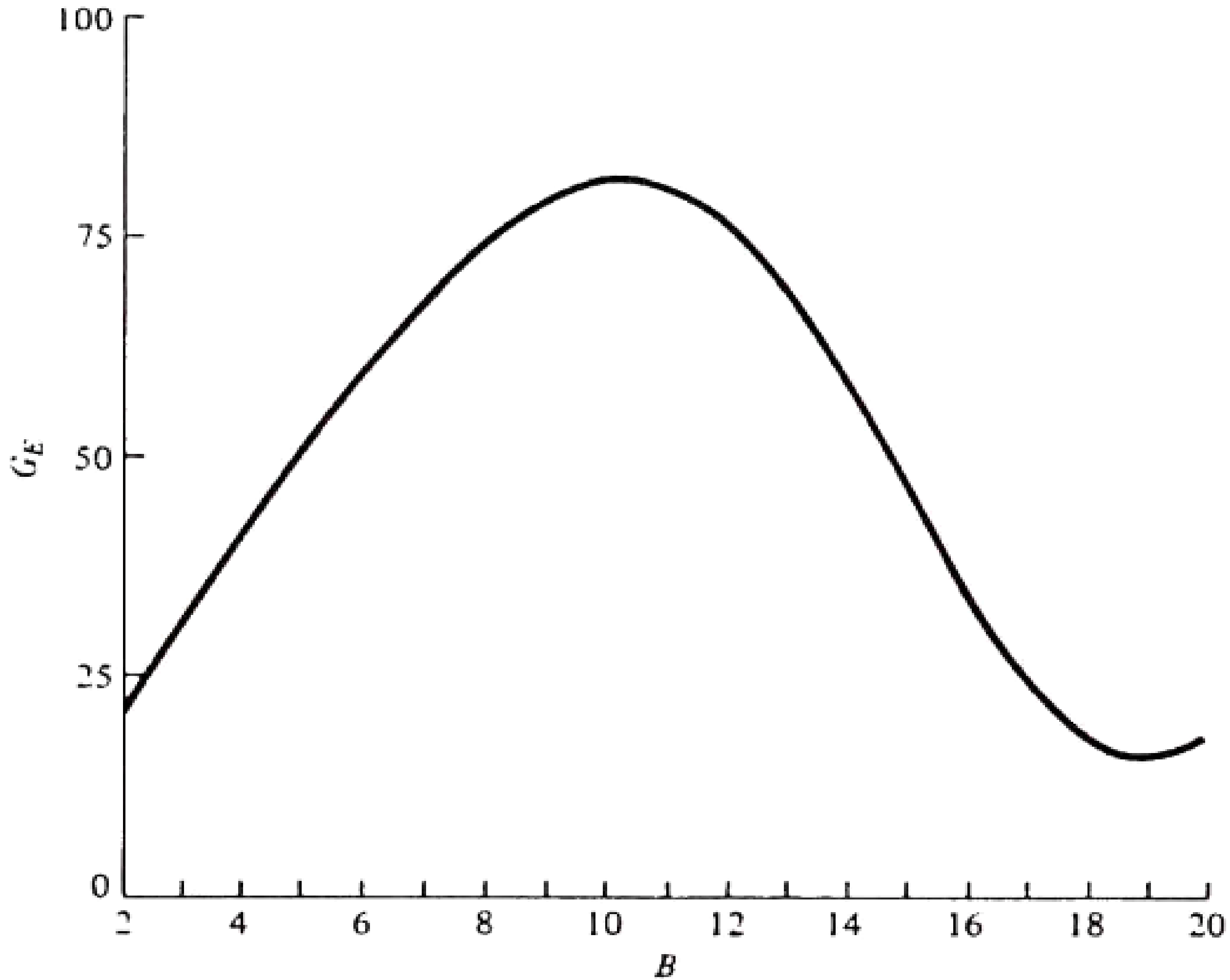
If $B < 2$, compute G_e using

$$G_E = \frac{32}{\pi}B$$

3. Calculate D_e

$$D_E = \frac{a}{\lambda}\frac{G_E}{\sqrt{\frac{50}{\rho_e/\lambda}}}$$

G_E as a Function of B



Example 13.3

Given: $a = 0.5\lambda,\ b = 0.25\lambda,\ b_1 = 2.75\lambda,\ \rho_1 = 6\lambda$

Compute the directivity

$$\rho_e = \lambda\sqrt{(6)^2 + \left(\frac{2.75}{2}\right)^2} = 6.1555\lambda$$

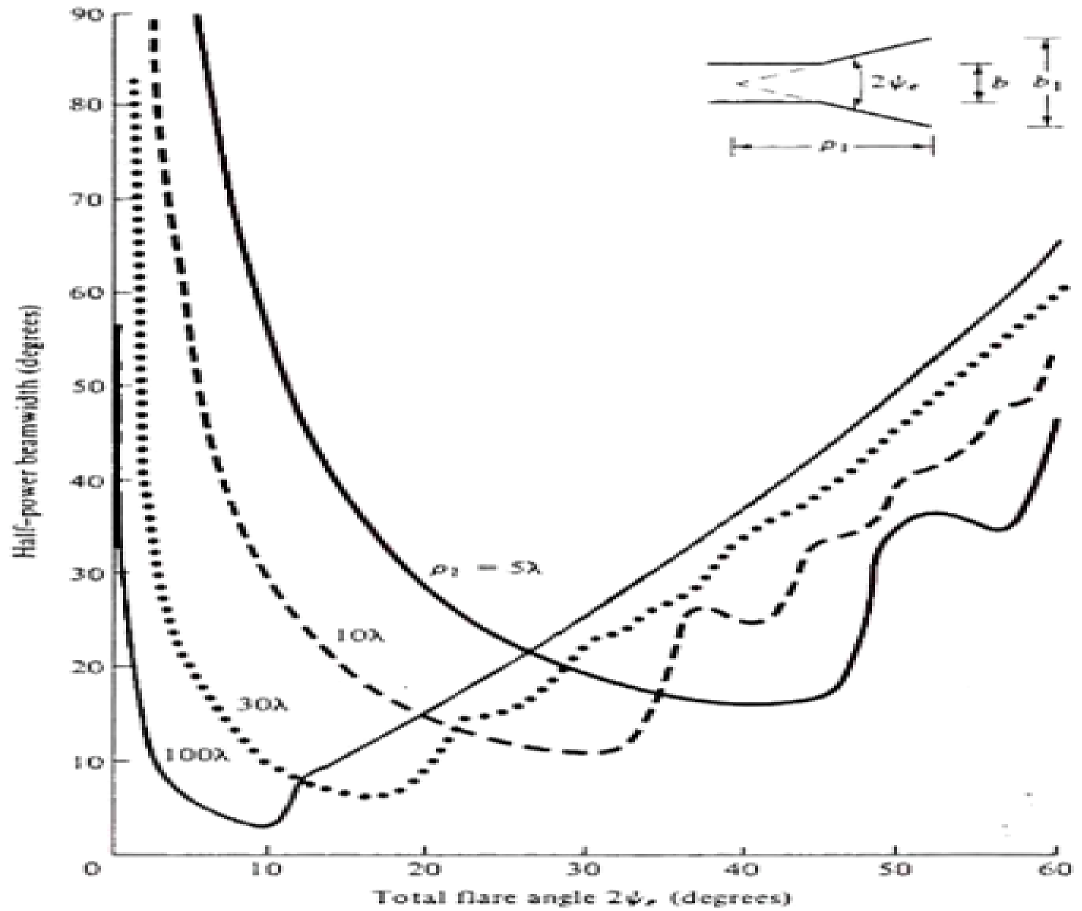
$$\sqrt{\frac{50}{\rho_e/\lambda}} = \sqrt{\frac{50}{6.1555}} = 2.85$$

$$B = 2.75(2.85) = 7.84$$

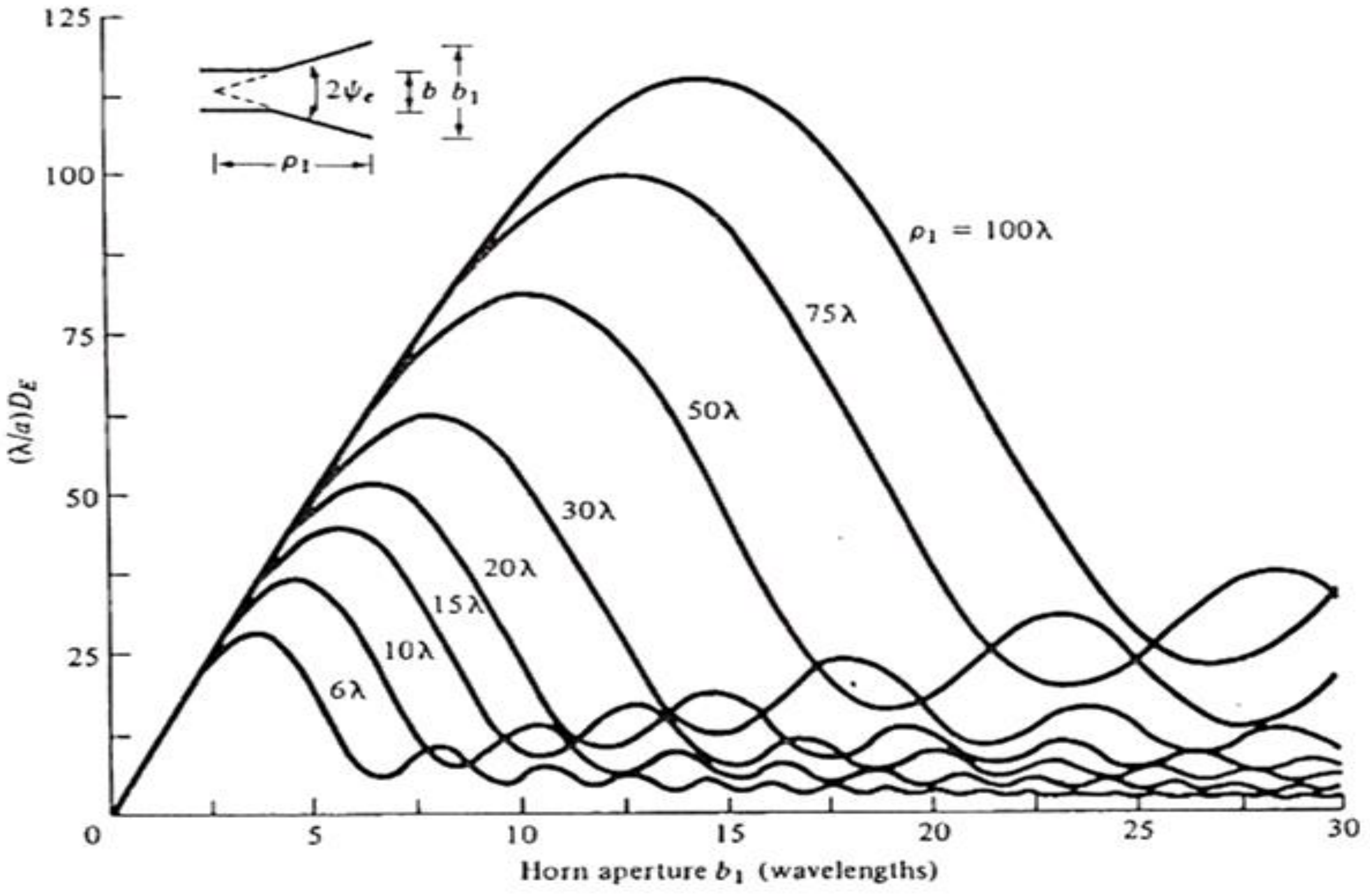
For $B = 7.84$, $G_E = 73.5$ from Figure 13.9. Thus, using (13-20c)

$$D_E = \frac{0.5(73.5)}{2.85} = 12.89 = 11.10 \text{ dB}$$

E Plane Horn HPBW



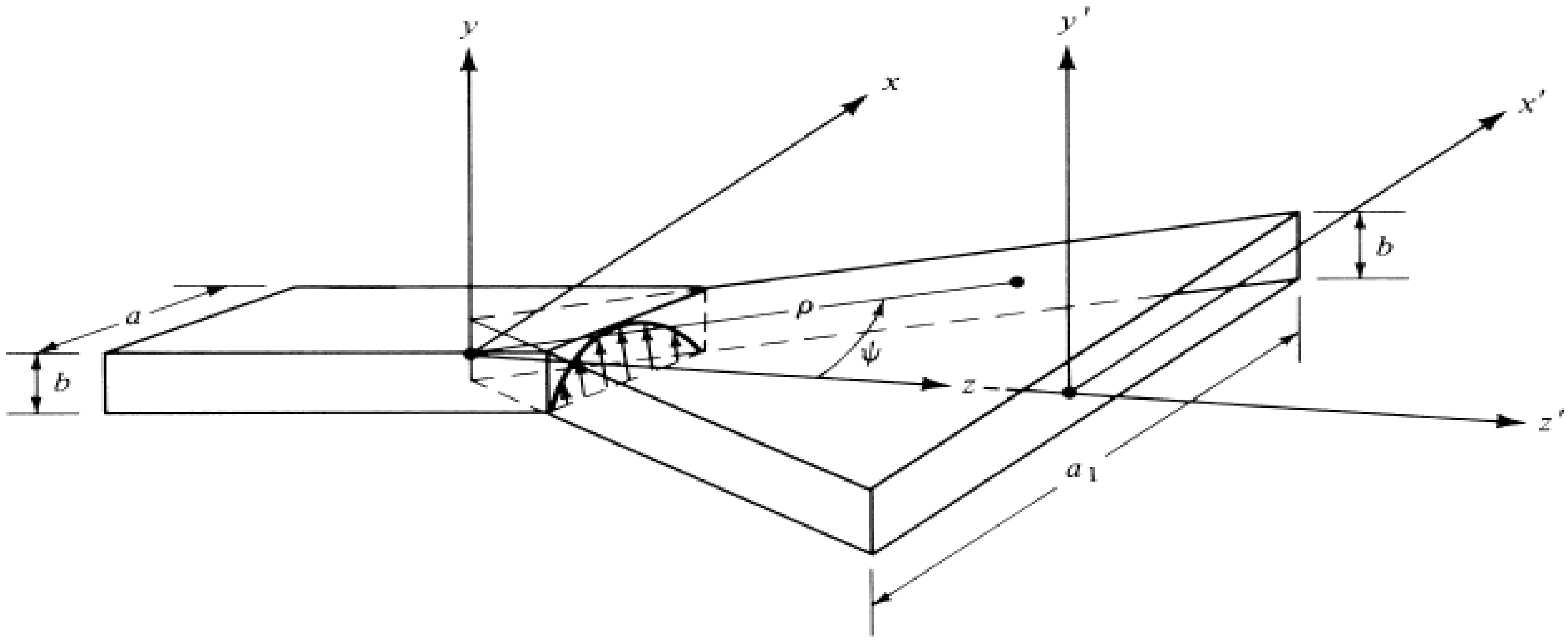
E Plane Horn Normalized Directivity



H Plane Horn

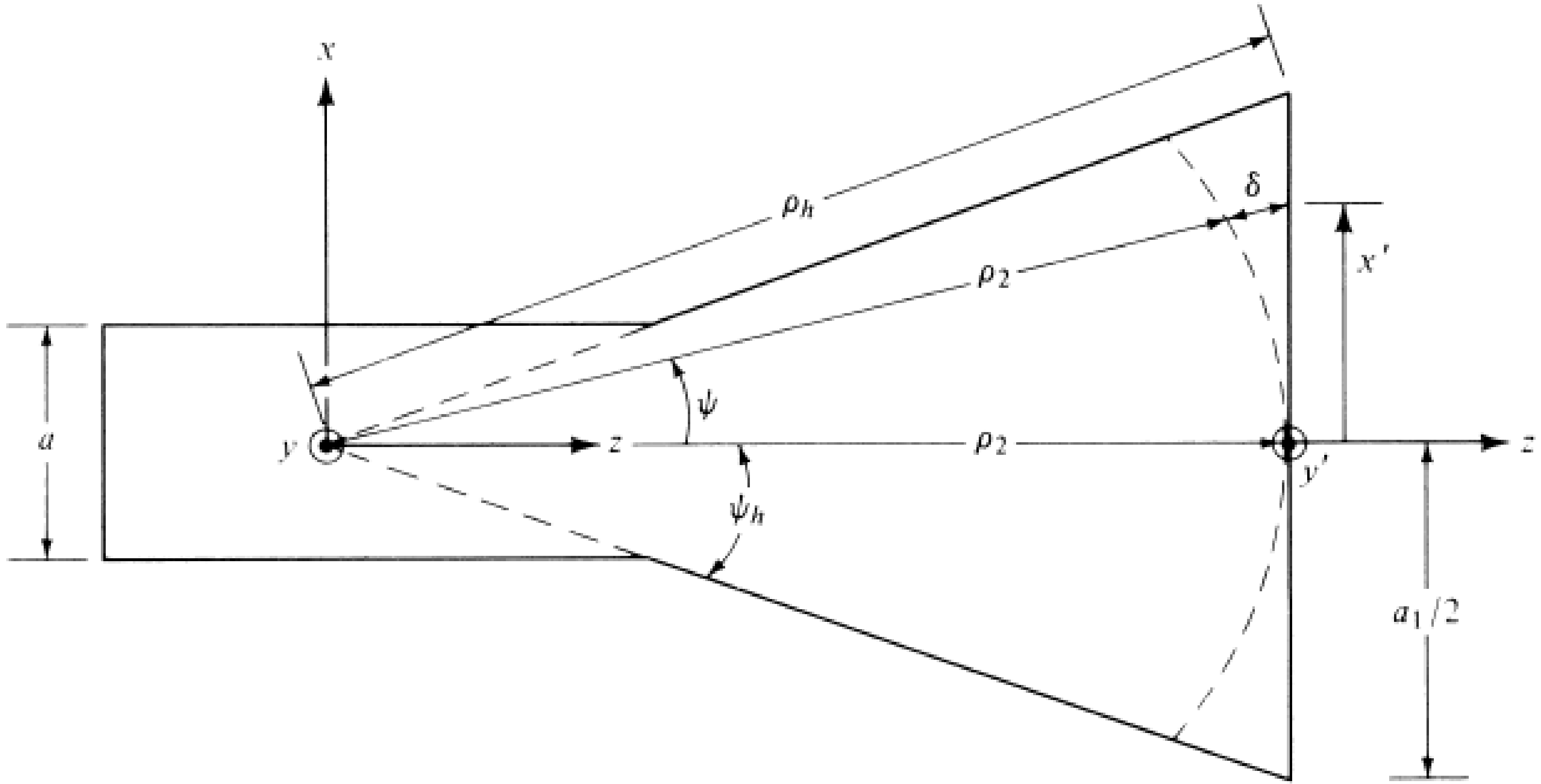
H-Plane Horn

Flaring the dimensions of a rectangular waveguide in the direction of the **H**-field.



(a) *H*–plane sectoral horn

H-Plane View



(b) *H*–plane view

Figure 13.10 *H*-plane sectoral horn and coordinate system.

H-Plane Horn

Aperture Fields:

$$E_y' = E_2 \cos\left(\frac{\pi}{a_1} x'\right) e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}\right)} \qquad (13\text{-}21b)$$

$$H_x' = -\frac{E_2}{\eta} \cos\left(\frac{\pi}{a_1} x'\right) e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}\right)} \qquad (13\text{-}21c)$$

$$\rho_2 = \rho_h \cos\psi_h \qquad (13\text{-}21d)$$

$$E_x' = H_y' = 0 \qquad (13\text{-}21a)$$

Radiated Fields

$$E_\theta = jE_2 \frac{b}{8}\sqrt{\frac{k\rho_2}{\pi}}\frac{e^{-jkr}}{r}$$

$$\cdot\left\{\sin\phi\,(1+\cos\theta)\frac{\sin Y}{Y}\cdot\left[e^{jf_1}F(t_1',t_2')+e^{jf_2}F(t_1'',t_2'')\right]\right\} \quad \text{(13-30b)}$$

$$E_\phi = jE_2 \frac{b}{8}\sqrt{\frac{k\rho_2}{\pi}}\frac{e^{-jkr}}{r}$$

$$\cdot\left\{\cos\phi\,(1+\cos\theta)\frac{\sin Y}{Y}\cdot\left[e^{jf_1}F(t_1',t_2')+e^{jf_2}F(t_1'',t_2'')\right]\right\} \quad \text{(13-30c)}$$

$$f_1 = \frac{k_x'^2\rho_2}{2k} \quad \text{(13-28b)}$$

$$Y = \frac{kb}{2}\sin\theta\sin\phi \quad \text{(13-28d)}$$

$$f_2 = \frac{k_x''^2\rho_2}{2k} \quad \text{(13-28c)}$$

$$F(t_1, t_2) = [C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)] \quad (13\text{-}28a)$$

$$t'_1 = \sqrt{\frac{1}{\pi k \rho_2}}\left(-\frac{ka_1}{2} - k'_x \rho_2\right) \quad (13\text{-}26a)$$

$$t'_2 = \sqrt{\frac{1}{\pi k \rho_2}}\left(+\frac{ka_1}{2} - k'_x \rho_2\right) \quad (13\text{-}26b)$$

$$k'_x = k \sin\theta \cos\phi + \frac{\pi}{a_1} \quad (13\text{-}26c)$$

$$t''_1 = \sqrt{\frac{1}{\pi k \rho_2}}\left(-\frac{ka_1}{2} - k''_x \rho_2\right) \quad (13\text{-}27a)$$

$$t''_2 = \sqrt{\frac{1}{\pi k \rho_2}}\left(+\frac{ka_1}{2} - k''_x \rho_2\right) \quad (13\text{-}27b)$$

$$k''_x = k \sin\theta \cos\phi - \frac{\pi}{a_1} \quad (13\text{-}27c)$$

E-Plane ($\phi = \pi/2$)

$$E_\theta = jE_2 \frac{b}{8}\sqrt{\frac{k\rho_2}{\pi}}\frac{e^{-jkr}}{r}$$

$$\cdot\left\{(1+\cos\theta)\frac{\sin Y}{Y}\left[e^{jf_1}F(t_1',t_2')+e^{jf_2}F(t_1'',t_2'')\right]\right\} \quad \text{(13-31b)}$$

$$Y = \frac{kb}{2}\sin\theta, \quad k'_x = \frac{\pi}{a_1} \quad \text{(13-31c)}$$

$$k'_x = -\frac{\pi}{a_1} \quad \text{(13-31d)}$$

$$E_r = E_\phi = 0 \quad \text{(13-31a)}$$

H-Plane ($\phi = 0$)

$$E_\phi = jE_2 \frac{b}{8}\sqrt{\frac{k\rho_2}{\pi}}\frac{e^{-jkr}}{r}$$

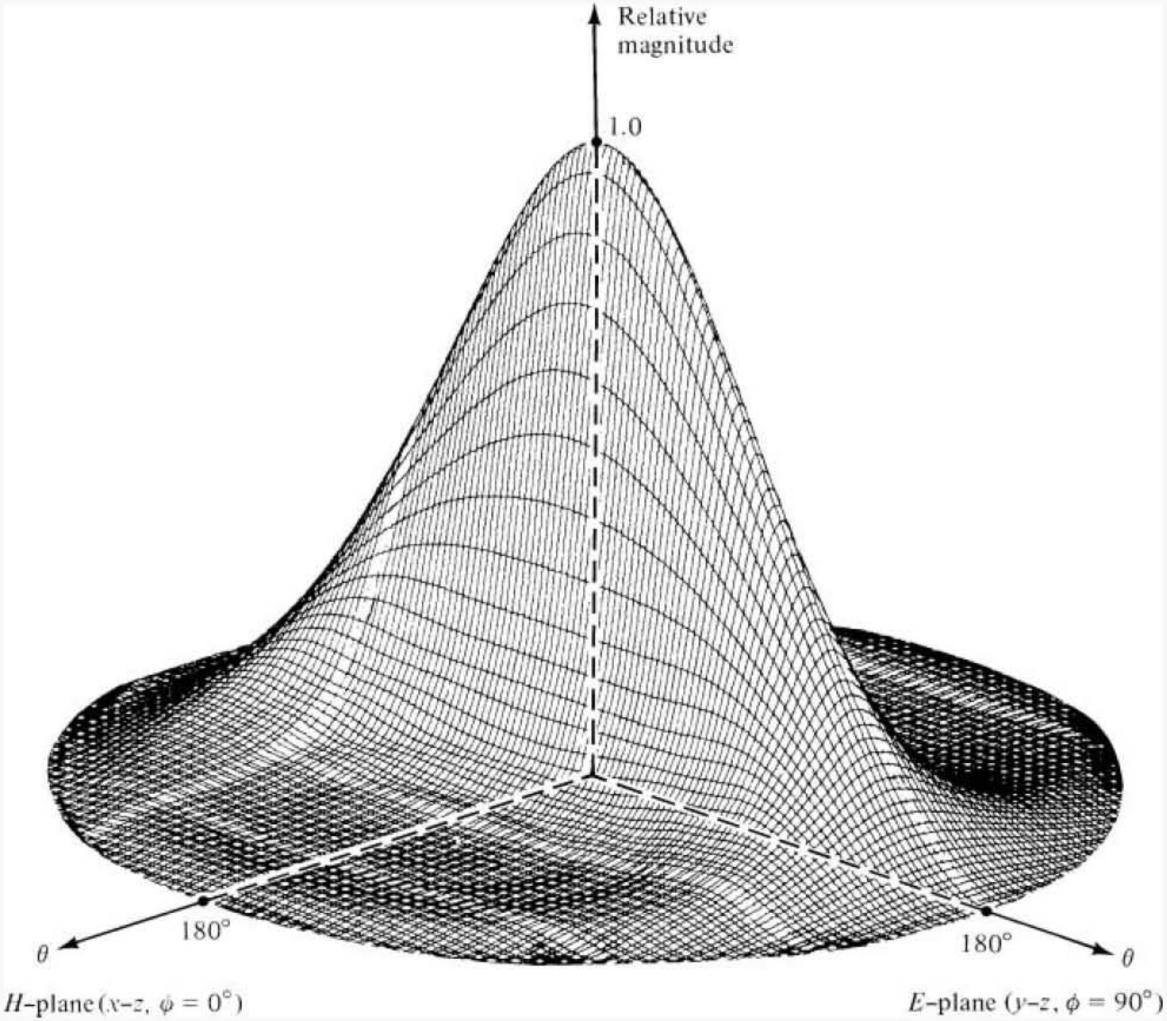
$$\cdot\left\{(1+\cos\theta)\left[e^{jf_1}F(t_1',t_2') + e^{jf_2}F(t_1'',t_2'')\right]\right\} \qquad (13\text{-}32b)$$

$$k_x' = k\sin\theta + \frac{\pi}{a_1} \qquad (13\text{-}32c)$$

$$k_x'' = k\sin\theta - \frac{\pi}{a_1} \qquad (13\text{-}32d)$$

$\rho_2 = 6\lambda$
$a_1 = 5.5\lambda$
$b = 0.25\lambda$

Fig. 13.11



$\rho_2 = 6\lambda$
$a_1 = 5.5\lambda$
$b = 0.25\lambda$

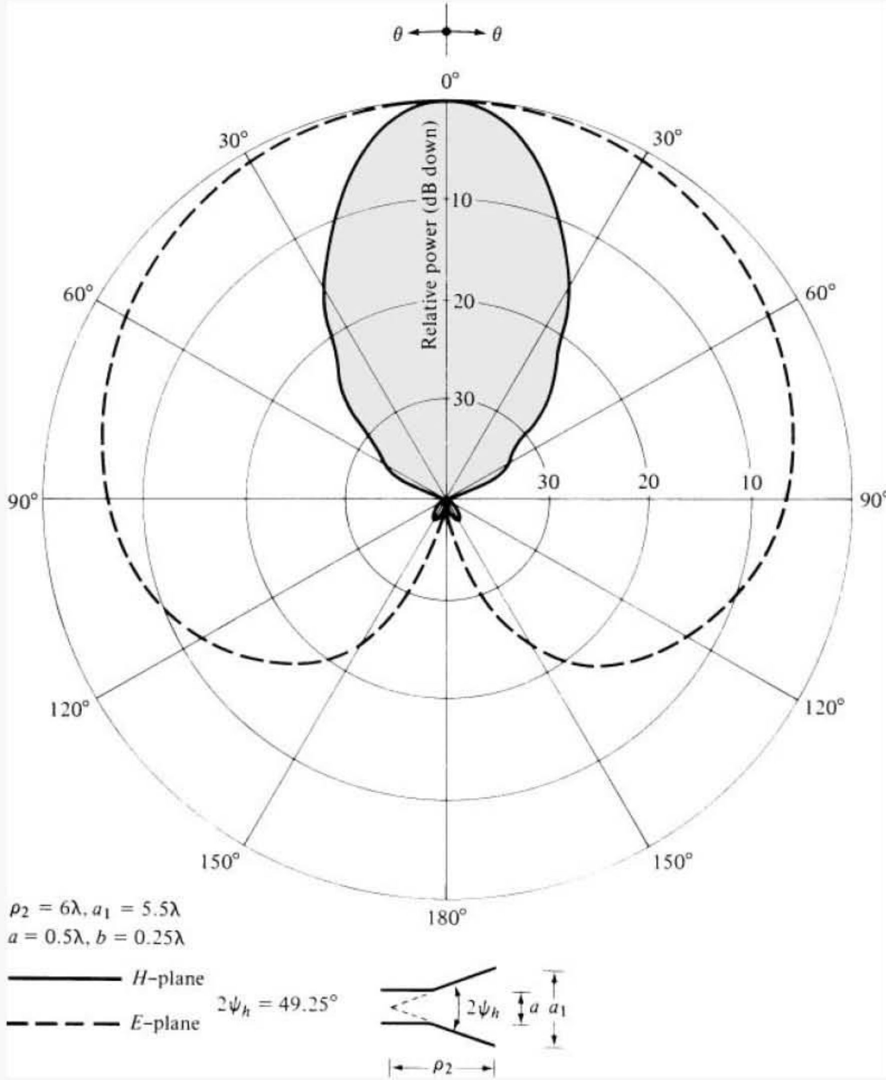
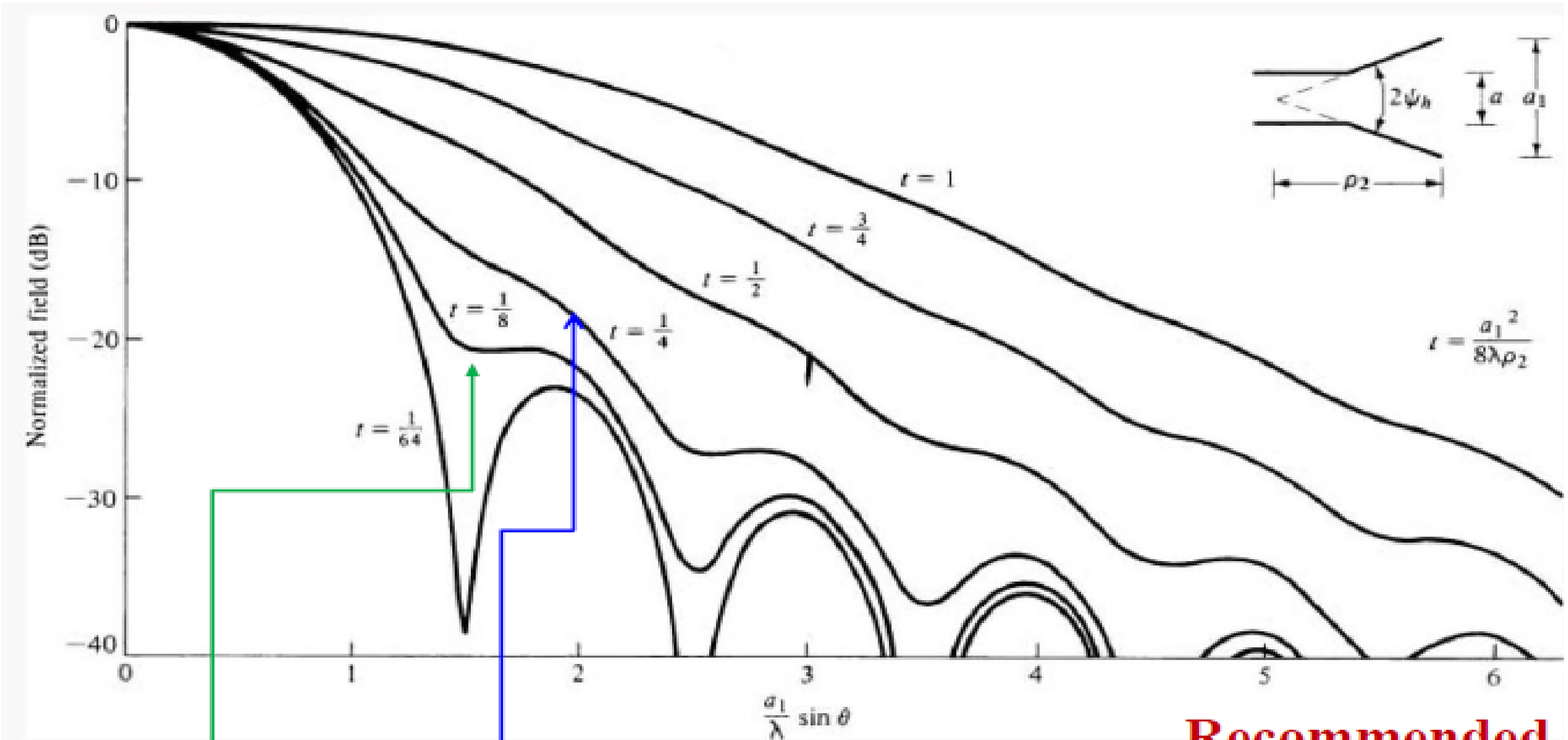


Fig. 13.12

H-Plane Sectoral: Universal Pattern



E-Field for t = 1/4 (δ_{max} = 90°)

E-Field for t = 1/8 (δ_{max} = 45°)

Recommended max. phase error between 45° and 90°

H-Plane Sectoral Horn: Max. Phase Error

Maximum Directivity occurs when

$$a_1 \simeq \sqrt{3\lambda\rho_2}$$

Maximum Phase error occurs when x' = a_1 / 2

$$\delta_{max} = 2\pi t, \text{ where } \quad t = \frac{a_1^2}{8\lambda\rho_2}$$

which gives 't' approximately equal to:

$$t_{op} = \frac{a_1^2}{8\lambda\rho_2}\bigg|_{a_1=\sqrt{3\lambda\rho_2}} = \frac{3}{8} \Rightarrow \delta_{max} = 135^\circ$$

Phase Error too high:

Not Recommended

Directivity

1. Calculate A by

$$A = \frac{a_1}{\lambda}\sqrt{\frac{50}{\rho_h/\lambda}}$$

2. Using this value of A, find the corresponding value of G_H from Figure 13.17. *If the value of A is smaller than 2, then compute G_H using*

$$G_H = \frac{32}{\pi}A \tag{13-42b}$$

3. Calculate D_H by using the value of G_H from Figure 13.17 or from (13-42b). Thus

$$\boxed{D_H = \frac{b}{\lambda}\frac{G_H}{\sqrt{\frac{50}{\rho_h/\lambda}}}} \tag{13-42c}$$

This is the actual directivity of the horn.

G_H As a Function of A

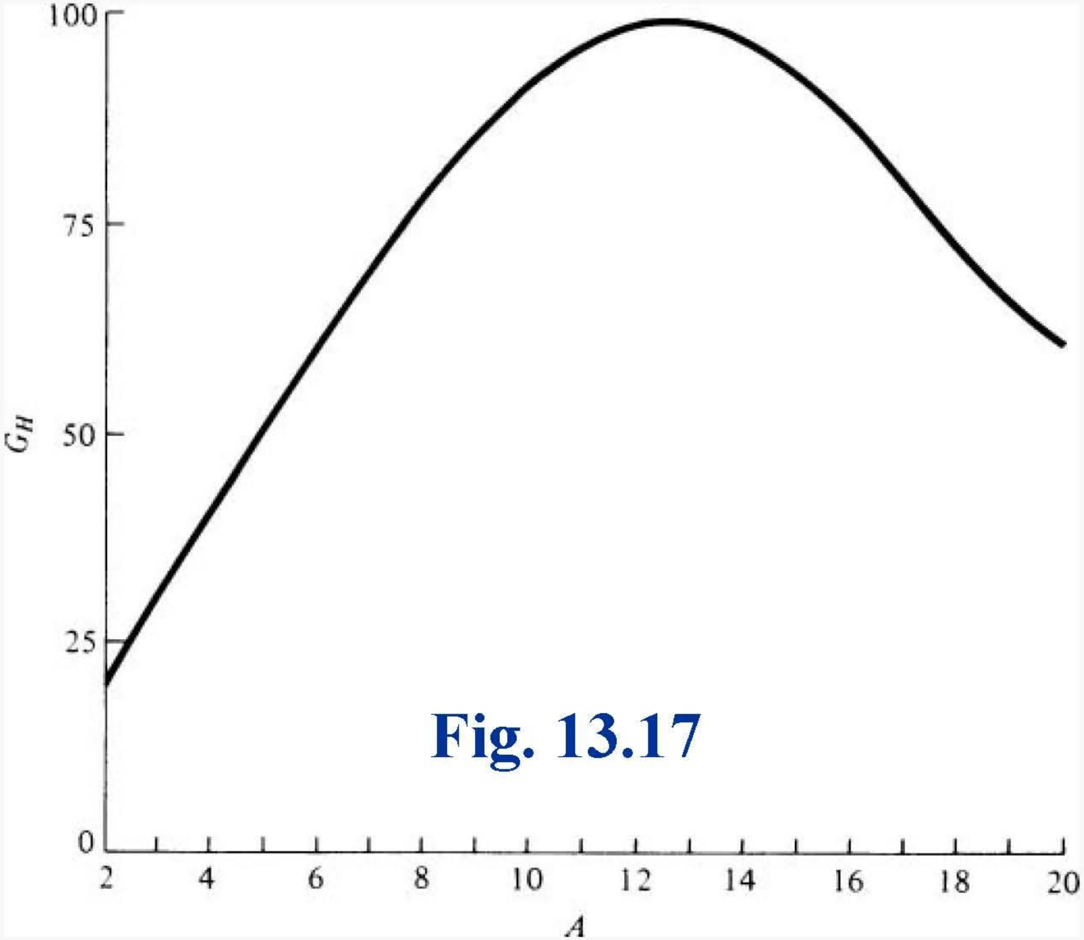


Fig. 13.17

Example 13.4

Given: $a = 0.5\lambda,\ b = 0.25\lambda,\ a_1 = 5.5\lambda,\ \rho_2 = 6\lambda$

Compute the directivity

$$\rho_h = \lambda\sqrt{(6)^2 + (5.5/2)^2} = 6.6\lambda$$

$$\sqrt{\frac{50}{\rho_h/\lambda}} = \sqrt{\frac{50}{6.6}} = 2.7524$$

$$A = 5.5(2.7524) = 15.14$$

For $A = 15.14$, $G_H = 91.8$ from Figure 13.17. Thus, using (13-42c)

$$D_H = \frac{0.25(91.8)}{2.7524} = 8.338 = 9.21\ \text{dB}$$